Accounts of Books. (1.) *Le grand & fameux Probleme de la* Quadrature *du* Cercle *reſolu Geometriquement par le Cercle & la Ligne droite, par Monſieur* Mallement *de* Meſſange. *A* Paris, *in* 12° 1686. *With a Refutation of the ſame, by Mr.* D. Cluverius. Reg. S. Soc.

THis Author is one of thoſe unhappy Geometricians, who without having acquired a through Underſtanding of the Principles, have yet thought themſelves able to maſter the abſtruſeſt Difficulties in this nice Mathematical Science, where the leaſt overſight or miſtake ſubverts the whole ſuperſtructure. Hence it is, that the true Quadrature of the Circle here pretended to, is loſt upon the ſame Rock with thoſe many others, which the leſs knowing and more opinionated of their own Skill have produced, in this and the laſt Century: But briefly to ſhew wherein the Paralogiſm of our Author conſiſts, we muſt firſt lay down the conſtruction, whereby he pretends to do the Buſineſs: In Tab. 2. Fig. 2. let $f\,k\,z\,k$ be a Circle, $f\,a\,z$ the Diameter, a the Center, $k\,z\,k$ an equilateral Triangle inſcribed, $B\,b$ a line equal to the three ſides of that Triangle, and dividing the Arch $f\,k$ equally in i, the line $i\,e$ will be half the ſide of a Hexagon inſcribed, which ſide taken 6 times, is the line $e\,E$ = to the circumference of the Hexagon; and dividing the Arch $i\,f$ in h, the ſine $h\,d$ is half the ſide of the Dodecagon inſcribed, and $D\,d = 24\,h\,d$ is the circumference of the Dodecagon; and proceeding after the ſame manner, the circumferences of Polygons of 24, 48, 96 ſides, *&c.* may be found, approaching ſtill nearer and nearer to the circumference of the Circle, which at length will be equal to the line $f\,F$ in the Tangent; but how to find the Point F is all the Skill: Here our Author tells us, that the Points $B\,E\,D\,F$ are all in the Arch of a Circle, whoſe center is in the line *f z* continued; but to contract his Work into a little compaſs, he aſſures us that it is all one, if inſtead of the whole circumferences *B b*, *E e*, *D d*, *F f* he take the lines *b q*, *e p*, *d o*, *f m*, each a third part of its correſpondent, and that in this caſe too, the Circle whoſe Center is *L*, taken in the Diameter *f z* continued, ſhall paſs through the Points *o*, *p*, *q* and interſect the Tangent *F f* in the Point *m*, ſo as to leave *f m* = to a third of the circumference of the Circle; which ſuppoſition being proved to be groundleſs and erroneous, all the conſequences drawn from thence muſt be ſo too. If our Author had but conſidered what the intermediate Points of the *Curve* between *o* & *p*, *p* & *q*, *q* & *t* ought to expreſs, he could not but have diſcovered the fallacy himſelf, for the lines *o d*, *p e*, *q b* are each proportioned to *f m*, as the ſines *k b*, *i e*, *h d* to their reſpective Arches *k f*, *i f*, *h f*, and ſo of all the reſt between *o* and *m*. This would have taught him, that the *Curve* he has occaſion to uſe, did univerſally expreſs the Proportion of the Arches to their Sines, by that of the line *f m* to its reſpective ordinates; that it was a ſort of *Linea Quadratrix*, to be reckoned among the *Lineæ Geometrice Irrationales*, or ſuch whoſe relation between the parts of the Diameter and the Ordinates, are not generally expreſſible by any one Equation; that this *Curve* did interſect the Circle in the middle of the Arch *k z*, and continge it in the point *z*. This *Curve* will be better underſtood in *Fig.* 3, where it is drawn as it ought, and wherein the proportion of the line *f m* to the lines *q b*, is as any Arch *k f* is to its Sine *k b*.

Tis evident, that this *Curve* is not the Circle *m o p q t* in *Fig.* 2, yet tis not apparent but that a Circle paſſing through the point *m*, may interſect it in ſeveral points, as *o*, *p*, *q*: (but to ſuppoſe it to paſs through all the extremities of the Circumferences of the infinite Polygons between the Circle and Triangle, or their thirds, is to make it coincident with that Curve.) It remains therefore to ſhew, that the Circle paſſing through *p* and *q*, whoſe center is in the Line *f z*, does not

paſs

paſs through the point o, which from the following Conſiderations will be made evident.

Firſt let it be required, by the extremities of the lines a,b,c, or b,g,f (in *Fig.* 4) parallel one to another and d,e, or k,l, given as parts or ſegments of the Axis or Diameter of the Figure, to determine what curvity paſſeth through their extremities, according to the conditions of the five Conical Sections. Firſt if it be found that $\frac{bb-aa}{de}\,\frac{cc+aa}{de+ee}$ is equal to $1 = \frac{cc-aa}{dd+ed}\,\frac{cc+bb}{ed}$, then it is the Characteriſtic of a Circle, the Lines a,b,c being diſpoſed in an uniform increaſing order: But if c the biggeſt ſtands in the middle, then $\frac{cc-aa}{dd+ed}+\frac{cc-bb}{ee+de}=1=\frac{cc-bb}{de}+\frac{bb-aa}{de+dd}$ will ſhew the ſame. If the Lines d, e be ſegments of a line drawn parallel to the *Axis*, then tranſpoſing and ordering the foregoing Equations, Rules alſo may be found accordingly. If $\frac{c-b}{b-a}=\frac{e}{d}$, or $\frac{c-a}{b-a}\,\frac{e}{d}=1$, then the Line paſſing through the Extremities is right: If $\frac{cc-bb}{bb-aa}=\frac{e}{a}$, then a Parabola is deſigned. The Characteriſtick of an Hyperbola or Ellipſis differs not from that of a Circle, but only by a Relation to the inequality of the *Axes*, and the alteration of the Signs $+$ and $-$.

Secondly, out of a, b, d, or b, c, e, lines given, that ſtand in the Arch of a Circle, to find the Diſtance from the Center $=$ to m, or $m+e$, and to determine the *Radius*. There is a little variety in the caſe, when the given lines are in the ſame Quadrant or otherwiſe: but there being only occaſion for this firſt Caſe, the Rule is this, $\frac{cc-bb}{2e}+\frac{1}{2}e=e+m$: And

$$\sqrt{\frac{cccc-2ccbb+bbbb}{4ee}}+\frac{1}{4}ee+\frac{1}{2}bb+\frac{1}{2}cc=\textit{Radius}.$$

Thirdly, in a Circle, having a, b, d and e, to find c: The

Equation is $\frac{b^2 e - a^2 e}{d} de - e^2 + b^2 = c^2$. Fourthly: To inſcribe Polygones in a continual double Progreſſion within a Circle, many different Rules may be given: the following will ſerve, which is the ſame with that, how to find the ſubtenſe of an Arch, out of the ſubtenſe of a double Arch. The Rule is thus; $2R^2 - \sqrt{4R^4 - A^2R^2} = B^2$; Suppoſing A to be the Chord of a double Arch, and B of a ſingle Arch. From hence it is eaſily deduced, that $\surd\, 3R^2$ being the ſide of an equilateral Triangle inſcribed, the ſide of a Hexagon will be R: of the Dodecagon $\surd\, un\, \frac{3}{2}R^2 - \surd\, \frac{1}{2}R^2$: and ſo for the reſt. Now reducing according to theſe Equations the Lines to Numbers, it will be found that in *Fig.* 2 *Tab.* 2

$bq = 173205,08$ $ab = 50000,00$
$ep = 200000,00$ $eb = 36602,54$ $fm = 209439,51$
$do = 207055,23$ $de = 9990,04$

But ſuppoſing, as our Authour will have it, that *d o* ſtands in the ſame Circle with *b q* and *e p*, it follows that the ſquare of *d o* = 42 2638679 &c. whereas it ſhould have been equall to 428718707 &c. Square of *d o* in the Table. The Square of the Tangent *f m* is alſo a great deal to ſmall, and the whole Quadrature too little: All which make it appear, that the Glory of *Lewis* the Great is not (as this Book pretends) much advanced by the Atchivements of this Author; who would have done well, in a Matter that ſo little needed it, to have forborn to make uſe of the ſacred Words of our Saviour, *Math.* 11*th.* 25*th.*

II. Voiage de Siam *des Peres Jeſuites envoyez par el Roy aux Indes & a la* Chine. *A* Paris 1686. 4°.

THIS is a ſecond Relation of the Voiage and Embaſſy of the *French* to the King of *Siam* in the Year 1685, and being a more particular Account than the former, an extract of this, 'twas thought, might ſuffice for both: That was compoſed by *le Chevalier Chaumont* the Embaſſadour, and now this by *le Pere Tachart* Jeſuite, who was one of ſix Fathers of his Order, that went with the Embaſſadour, as Miſſionaries to *China*. The whole being much interſperſed with matters of Religion and Ceremony, I ſhall only take notice of ſuch things as relate to Arts and Sciences, and particularly of the Aſtronomical Obſervations made at the *Cape* of *Good Hope* and at *Siam*; whereby the Longitudes of thoſe places are ſtated: following herein the Authors method.

He divides his Treatiſe into ſix Books, whereof the firſt contains the Voiage from *Breſt* to the *Cape* of *Good-Hope*. Here he gives the reaſons and motives of ſending this Embaſſie, as likewiſe the ſix Jeſuites who are Mathematicians and by the Kings Letters Patents are ſo ſtiled; Their Inſtructions being, beſides their Spiritual Function, to proſecute the buſineſs of the *Royal Academie* of *Paris* (of which they are admitted Members) by accurately obſerving the curious things in Art and Nature, and particularly to make Obſervations for diſcovering the *Longitudes* of the Places where they paſs; for which purpoſe they are well provided with Inſtruments. They ſailed from *Breſt* on the third of *March ſt. n.* and arrived at the *Cape* of *Good-Hope* the laſt day of *May*, taking notice by the way of the ſeveral remarkables in that Voiage, which are here too well known to need repeating; But mentioning the faultineſs and rectification of the Southern Conſtellations, our Author is not willing to take notice of what has been done in that matter by a member of of the *Royal Society* of *London*, tho' his Catalogue of thoſe Stars hath been tranſlated from *Latine* into *French* and Printed at *Paris*, and an Account thereof is in the *Journal des Scavans* of *Aug*. 7. & *Sep*. 4. 1679; but ſpeaks of it as a thing not done, wiſhing they had had the opportunity to augment the Science of *Aſtronomy*, by obſerving them themſelves.

The ſecond Book is entituled the Voyage from the *Cape* of *Good-Hope* to the Iland *Java*; but is chiefly taken up with the deſcription of the *Colonie* of the *Hollanders* there, the Natives, and the *Aſtronomical* Obſervations they made there during their ſtay, by which they have determined the Longitude of the *Cape* of *Good-Hope* 18 degrees to the *Eaſt* of *Paris*: (but we here muſt begg leave to make a Remark) He mentions 7 ſeveral Nations of the Natives, *viz*. the *Hottentots*, whom he deſcribes at large, the *Namaquaas*, (of theſe two there are the Figures, the *Ubiquaas*, the *Gouriquaas*, *Ilaſſiquaas*, *Souſiquaas* and the *Odiquaas*: and here he relates a Voiage made in the Year 1685, as farr as the *Tropick*, by the Governour of the *Cape*, Mr. *Vanderſtell*; who is ſaid to have found about the Latitude of 27 degrees and about 10 or 12 leagues from the ſhore, a Nation of Natives that are very Muſical, who have long Hair flowing on their ſhoulders, ſome of the Men as White as *Europeans*, and their Women Naturally very White, but they Blacken themſelves to pleaſe their Huſbands: This Nation ſeems to have much more Intelligence than their Neighbours, but ſome circumſtances ſeem to argue it Fabulous.

Here are likewiſe the Figures of a *Stagg* with Horns like a *Goat*, of the *Zembra*, the *Sea Cow*, the *Ceraſtes* or Horned *Serpent*, the *Camelion*, and two ſorts of uncommon *Lizards*, whereof one is made to have 3 fair Croſſes on his back: Speaking of *Elephants* he ſays he was told by creditable perſons, that they had ſeen the foot ſteps of *Elephants* two foot and half diameter; and that there are *Rhinocerotes* there as bigg as ordinary *Elephants*, but, by I know not what miſtake, he makes the *Rhinocerote* a two horned Animal. The Voiage

age from the *Cape* to *Jaua* (in ſight whereof they arrived *Aug. 5th.ſt. n.*) contains nothing very Extraordinary.

The third Book is the Voiage from *Java* to the Kingdome of *Siam:* which is chiefly taken up with what occurred at *Bantam* and *Batavia*, and at their arrival on the Coaſt of *Siam*, here are repreſented the Roads of *Bantam* and *Batavia*, together with the Plan of the City and Fortreſs of *Batavia*.

The fourth Book deſcribes the Entry and Audience of the *French* Embaſſadour at the Court of *Siam*, who, as they ſay, was received with more Honour and Reſpect, than was ever yet ſhewn to any Ambaſſadour whatever; and that even thoſe of *Perſia*, the *Mogul* and the *Tartar* Emperour of *China*, (tho' his neighbour, and by much the moſt Potent Monarch of the Univerſe) preſent themſelves before the King of *Siam* on their Knees, whereas Mr. *Chaumont* the *French* Ambaſſadour made his harangue, ſitting with his Hat on his Head. Here are deſcribed the *Baloons* or *Barges* of State which are uſed at *Siam*, which are of a very odd Figure, as of *Serpents* or *Sea-Horſes*, but which by their ſharpneſs and number of Oars are of an incredible Swiftneſs: here likewiſe 'tis related that the old White *Elephant* of the King of *Siam* is near upon 300 Years Old; as alſo that there are *Tumblers* there, of an extraordinary Agility, as that they would ſtand upon one Foot on the top of a *Bamboo* of 80 or 100 Foot high, and then turn themſelves, and ſtand on their Heads thereon, and afterwards hang, by the Chin only, on the top of the ſame, and then deſcend by a Ladder down right, with an incredable Swiftneſs, working their Bodies all the while through the Rounds of the Ladder.

The fifth Book is entituled the Return of the Voiage of *Siam*, and firſt relates ſeveral notable Shews preſented to entertain the Ambaſſadour; as the fight of two *Elephants*, who were only ſuffered to twiſt each others Teeth, as Bulls do their Horns; the fight of an *Elephant* and a *Tiger* or *rather a Panther according to the deſcription*; and the manner of catching the wild *Elephants*, by alluring them into an incloſure by the means of a Female tame *Elephant*, and the like. Next are related ſeveral Obſervations of the variation of the Magnetical Needle, which was found towards the end of the Year 1685, to be about half a degree Weſt, at *Louvo* near *Siam*; as likewiſe the Obſervation of an Eclipſe of the Moon on the 10*th*. *December ſt. n. poſt mediam noctem*, made at a place near *Louvo* called *Thlee-Pouſſonne*, in the preſence of the King. It begun about 15*h*. 20*m*. the total Darkneſs at 16*h*. 23*m*. 45*ſ*. the Emerſion or end of Total Darkneſs at 18*h*. 2*m*. 36*ſ*. or rather, as is there ſaid, at 18*h*. 10*m*. 25*ſ*. whereby the Longitude of this place is found 98 degrees and half from *Paris*, and about 6*h*. 45*m*. to the Eaſt of *London*, as may be ſeen by comparing this Obſervation with the Obſervations thereof made at *Dantzick*, *Nurenburg* and *Liſbon*, Publiſhed in *Philoſ. Tranſact*. Num. 178, 182 and 184. And whereas tis here ſaid that ſome Charts have made the Longitude of *Siam* above 20 degrees more than it is, tis to be underſtood only of the Charts of *Sanſon*, which in this particular are the worſt extant: But that this Correction is juſt, we are fully ſatiſfied, by the like errors in thoſe parts, diſcovered and publiſhed in the *Philoſoph. Collect*. of *Feb*. 168½, and in *Philoſ. Tranſ*. of *June* 1683. The relation of the homeward bound Voiage (which was of about 6 Months) is ſhort and contains very few conſiderable Remarks.

The ſixth and laſt Book is of the manners and Religion of the *Siammers*, where is a ſhort deſcription of the Kingdome and Capital City of *Siam*: Next the habits of the People, and their uſe of *Betele*, *Arek*, and *Tea* is deſcribed, as likewiſe the Root *Ginſeng* ſo much eſteemed in the Eaſt, with its Vertues and Uſes. As to the Religion of this people, which is here deſcribed at large, I ſhall ſay little to it, as not falling under our Argument, only one principal point therein is the *Metempſychoſis* of *Pythagoras* and the *Bramines*, and they hold the Eternity of the World, but on the contrary they ſuppoſe God Mortal, Corporeal and produced in time: their preſent God they call *Sommonokhodom*.

A Remark concerning the Longitude of the Cape *of* Good Hope.

IN the ſecond Book of this Voiage are related two Obſervations of the *Satellites* of *Jupiter*, capable, if well made, to aſcertain the Longitude of the *Cape* of *Good Hope*. The firſt was there made *June 2d. ſt. n.* 1685, when at 11*h.* 29*m.* 20*ſ.* the firſt or innermoſt *Satellite* touched the *Weſtern* edge of *Jupiter* and at 11*h.* 30*m.* 50*ſ.* it appeared no more: this Obſervation is ſaid to be made with an excellent *Teleſcope* of twelve Foot: The other was on *June 4th.* following *ſt. n.* when the *Emerſion* of the ſame *Satellite* was obſerved at 9*h.* 37*m.* 40*ſ.* from which latter is concluded, that the Longitude of the *Cape* is 18 *gr.* to the Eaſt of *Paris*, for that the ſaid *Emerſion*, according to the *Calculus* of Sigr. *Caſſini*, in the Meridian of *Paris* ought to have happened at 8*h.* 26*m.* This ſame *Emerſion* is computed by Mr. *Flamſteed* at 8*h.* 19*m.* at *London*, that is, 3*m.* later than by Sigr. *Caſſini*; and conſidering that neither is verified by Obſervation in *Europe*, the Longitude hence deduced is doubtful at leaſt 3 *minutes*, if this had been the only Obſervation: But the former being conſidered will yet ſhew that there is a much greater doubt ſtill remaining: For from certain *Aſtronomical* principles the *parallax of the Orb*, or difference between the place of *Jupiter* ſeen from the *Sun* and *Earth* was, at the time of the firſt Obſervation, 9*gr.* 19*m.* which Arch that *Satellite* moves in 1*h.* 6*m.* and the utmoſt duration of an Eclipſe thereof in this poſition of *Jupiter* being ſcarce 2*h.* 20*m.* (as appears by the accurate Obſervations of Mr. *Caſſini* and Mr. *Flamſteed*) it will follow, that from the Immerſion behind *Jupiters* Weſtern Edg, to the Emerſion out of the ſhaddow, there could not be full 3*h.* 26*m.* wherefore the Emerſion out of the ſhaddow, on *June 2d.* ought, according to the time of the Immerſion, to be at 14*h.* 56*m.* at lateſt, at the *Cape*; which by Mr. *Flamſteeds Calculus* was at *London* 13*h.* 51*m.* or according to Sigr. *Caſſini* at

13*h*. 58*m*. at *Paris*. Hence the Longitude of the *Cape* will be found but 14 *degrees* and half at moſt to the Eaſt of *Paris*; ſo that theſe 2 Obſervations will differ in the reſult about a quarter of an hour; which is a little too much. However there are ſome reaſons that ſeem to argue for this latter Longitude rather than the former; for it is much eaſier to obſerve what becomes of a luminous Object that appears, than to wait upon the firſt appearance of a Star Eclipſed: and tis probable that the *Satellite* might, in the latter time, be ſeveral minutes Emerged out of the ſhaddow, when they might firſt perceive it; but they could not but ſee the application to the Body of *Jupiter* in the former, if we may ſuppoſe their *Teleſcopes* ſo good as they are ſaid to be: And that the *Cape* of *Good Hope* is not more than an hour to the Eaſt of *Paris*, is proved by the conſtant conſent of our Navigators, who find by their Reckonings that the Iſland of St. *Helena* is about 22 or 23 degrees of Longitude to the Weſtward of the *Cape*: (and that Sailing both backward and forwards tis the ſame, which takes away the Objection of Currents) now by accurate Obſervations made at St. *Helena*, and compared with others made in *Europe* at the ſame time, the Longitude of that Iſle is certainly about 8 ½ degrees to the Weſt of *Paris*: It follows therefore that the *Cape* cannot be much more than 14 or 15 degrees to the Eaſt of *Paris*; and undoubtedly it muſt be leſs than 18, for 3 degrees is much too great an Errour to be committed in ſo ſhort a diſtance Sailing.

N.185.

Tab. II

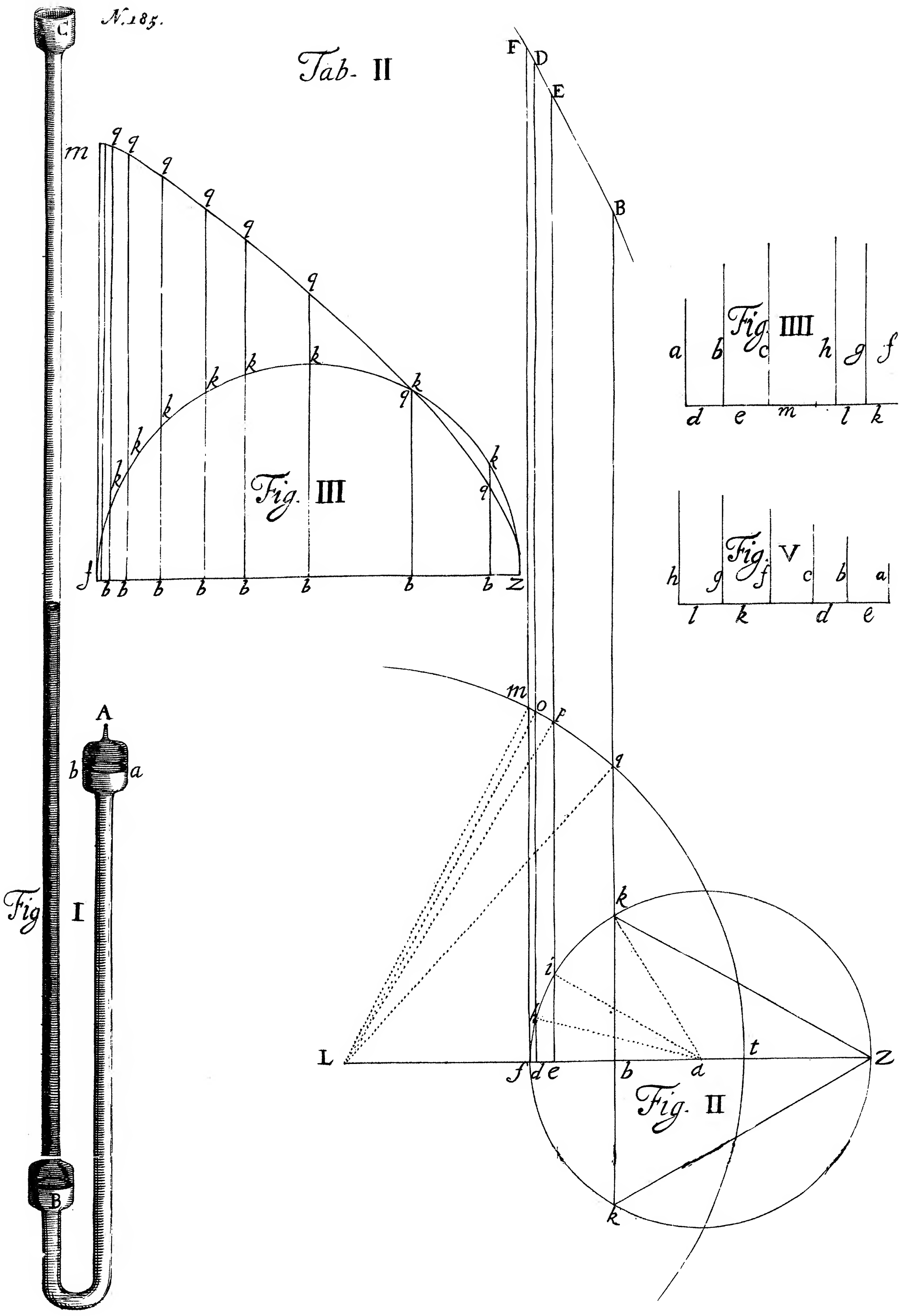

Fig I

Fig. III

Fig. II

Fig. IIII

Fig. V

N. 185.